INTEGRATED ELD — MATHEMATICS

Grade-level content instruction in mathematics (including pre-algebra, algebra, geometry, statistics, pre-calculus/trigonometry, and calculus in the upper grades) with language support for ELs (e.g., content-related discourse patterns and problem-solving and justification) that is implemented throughout instruction.

Abedi, J., & Herman, J. (2010). Assessing English Language Learners' opportunity to learn mathematics: Issues and limitations. *Teachers College Record*, *112*(3), 723–746.

The authors considered ELL opportunities to learn (OTL) in mathematics while also investigating factors that influence differences in levels of OTL for ELLs and non-ELLs. Looking at 24 8th grade Algebra I classrooms (students: n = 602, teachers: n = 9) at three urban middle schools in a large Southern California school district, the authors drew on a number of data sources, including teacher and student surveys of OTL, an assessment on initial Algebra I content knowledge, the fluency subscale for the Language Assessment Scale, and a student background questionnaire that provided insights on the mathematics preparation of students as well as their understanding of teacher directions. Study results indicated that: (1) measures of classroom OTL were associated with student performance, (2) compared to non-ELLs, ELLs reported a lower level of OTL, (3) a high concentration of ELLs in a classroom was associated with lower levels of OTL, and (4) effective access to OTL appeared to be influenced by English proficiency and self-reported ability to understand teachers' instruction. These findings illuminate the limits of looking at exposure, as exposure alone does not ensure adequate access to curriculum or appropriateness of learning opportunities.

SOURCE: journal article *TYPE*: empirical
KEY WORDS: opportunities to learn, middle school, academic language, mathematics

Arizmendi, G. D., Li, J-T., Van Horn, M. L., Stefania D. Petcu, S. D., & Swanson, H. L. (2021). Language-focused interventions on math performance for English Learners: A selective meta-analysis of the literature. *Learning Disabilities Research & Practice*, *36*(1), 56–75. https://doi.org/10.1111/ldrp.12239

Math is often thought of as being free from language. For all learners however, math requires proficiency in math vocabulary which poses an additional challenge for ELs. In this study, Arizmendi and her colleagues focused on math interventions that target the development of math language. To conduct their investigation, Arizmendi et al. conducted a meta-analysis of studies that met the following criteria: (1) empirical studies published in peer-reviewed journals; (2) with a focus on interventions for increasing ELs math performance by focusing on the development of math language; (3) with reported information on EL participants' performance on standardized or experimental math measures; (4) that utilized either a treatment and a control group with pre- and post- test data or a single-subject design (treatment only) with at least three data points; and (5) that they reported measures of treatment fidelity. They found a total of 12 studies that met their criteria and of them, six followed a group-study design with treatment and control groups with pre-and post-test measures, and six followed a single-subject

design where the treatment group was tracked over time. Across all 12 studies in the meta-analysis, Arizmendi et al. found that all of the interventions evaluated in the studies were delivered in English (one of the interventions was delivered in Spanish). They also identified 18 language strategies woven into interventions for increasing ELs math performance: the most commonly used language strategies across the studies including promoting math communication, building math vocabulary, scaffolding language, monitoring comprehension of terms, using everyday language, and reducing linguistic complexity. In terms of effect size, group-study design studies had a mean of g=0.26 (which according to Cohen's criteria is small) and single-subject design studies had a PND mean of 81.01, and a Phi coefficient of .66 (which is considered a moderate to large effect size). They further found the following four moderators that yielded statistically significant findings: (1) interventions delivered in kindergarten had higher effect sizes than those delivered in middle school; (2) traditional interventions yielded larger effect sizes than computer-based interventions; (3) interventions focusing on numeracy skills yielded higher effect sizes than mixed math skills; and (4) interventions between 26 and 36 weeks yielded larger effect size than those between 11 and 25 weeks. Arizmendi and her colleagues used these findings to present implications for more clearly defined and tested math interventions that would facilitate, for example, an investigation of the effect size of specific program components and instructional strategies on ELs.

SOURCE: journal article *TYPE*: empirical
KEY WORDS: meta-analysis, mathematics achievement, English language learners, intervention, kindergarten

de Araujo, Z., Roberts, S. A., Willey, C., & Zahner, W. (2018). English Learners in K–12 mathematics education: A review of the literature. *Review of Educational Research, 88*(6), 879–919. https://doi.org/10.3102/0034654318798093

In this review, de Araujo and colleagues analyze literature published between 2000 and 2015 on the topic of mathematics teaching and learning with K–12 ELs. Using KEY WORDS suggested by prior scholars in the field, the researchers searched the ERIC database and identified 75 peer-reviewed empirical studies on mathematics for K–12 English learners. They employed an iterative approach to organizing the literature and, in its final presentation, organized the literature by focus: learning, teaching, and teacher education. The researchers synthesize the results of the studies in their literature review through a sociocultural perspective on language in mathematics. Themes that focus on learning include: the relationship between mathematics performance and language proficiency, ELs' use of linguistic resources to learn mathematics, and ELs' use of cultural resources in mathematics learning. Themes that focus on teaching include: teachers' histories, teaching contexts, and teachers' practices, which include practices that promote access to content and practices that support mathematics discourse. Themes that focus on teacher education include pre-service and in-service teacher education. Two recommendations are (1) Expand research on mathematics teacher preparation programs for teaching ELs, and (2) Diversify the types of evidence collected in research on mathematics teaching and learning with K–12 ELs beyond qualitative.

SOURCE: journal article *TYPE*: review
KEY WORDS: mathematics education, mathematics teaching, bilingual learners, English learners

Dominguez, H. (2012). ***The making of a meaning maker: An English Learner's participation in mathematics*****. North American Chapter of the International Group for the Psychology of Mathematics Education. https://files.eric.ed.gov/fulltext/ED585055.pdf**

The purpose of this paper was to examine how English learners engage in and make meaning of mathematics instruction. To conduct the study, Dominguez videotaped math instruction in one fourth grade classroom, four times a week for one school year. One English Learner in the classroom was the focus of the study. Findings showed that the process of creating a meaning maker is arduous and includes moments of resistance as the student learned to participate in mathematics discussions from responding only when called on by the teacher and responding to the teacher, to engaging in math talk with peers and sharing understandings and strategies. The author highlighted the importance of re-envisioning student participation in mathematics not as a single act but as a process that evolves along a continuum.

SOURCE: journal article *TYPE*: empirical
KEY WORDS: meaning making, participation, mathematics, English learners, elementary school education

Gutiérrez, R. (2018). Political ***conocimiento*** **for teaching mathematics. In S.E. Kastberg, A.M. Tyminski, A.E. Lischka & W.B. Sanchez (Eds.),** ***Building support for scholarly practices in mathematics methods*** **(pp. 11–38). Information Age Publishing.**

In her work, Gutiérrez argues that unlike popular belief, mathematics instruction is not straightforward, universal, and culture free but that equity in mathematics education and the ability for mathematics teachers to reach English Learners and other marginalized groups involves teachers' ability to negotiate the politics in their practice. Gutiérrez draws from recent developments that attempt to privatize public schools and from the literature to make her case that all teaching, including mathematics teaching, is heavily sociopolitical. She argues that who gets credit for developing mathematics, being capable of mathematics, or seen as part of the mathematics community is generally White. Her theory of political *conocimiento* is the idea that it helps to deconstruct deficit narratives in society about students, teachers, or public education and better prepares teachers to question policies around them and use their professional judgement when making decisions about the learning opportunities that they create and experience alongside their students. Gutiérrez argues that teacher education programs can develop political knowledge in pre-service teachers. She proposes a diagram with four dimensions of equity/learning to include: (1) access, (2) achievement, (3) identity, and (4) power —all held together by *Nepantla*, a Nahua metaphysics that recognizes and values opposing forces. Her theory argues that teachers must recognize and live in tension and then decide how best to act for the benefits of marginalized students including Black, Latino, American Indian, newcomers, and emergent bilinguals. Gutiérrez proposes two activities for the development of political *conocimiento* in pre-service teachers, namely, The Mirror Test and In My Shoes.

SOURCE: book chapter *TYPE*: theoretical
KEY WORDS: critical professional development, politics of teaching, mathematics education, urban education

Lewis, B., King, M. S., & Schiess, J. O. (2020). ***Language counts: Supporting early math development for dual language learners*****. Bellwether Education Partners.**
https://bellwethereducation.org/sites/default/files/Bellwether_LanguageCounts-HSF_Final.pdf

This report explores effective strategies to promote early math learning for Dual Language Learners (DLLs) –defined as children 0–8 living in households where at least one parent speaks a language other than English. Lewis and his colleagues review the literature on evidence-based practices across three areas: math instruction for early learners, math instruction for older ELs, and engaging families of DLLs and ELs in supporting children's home learning. They profile two organizations that exhibit the evidence-based practices for supporting DLLs' early math achievement; Mighty Math –a parent peer leadership model that raises awareness among families of DLLs about promoting early math at home; and Zeno Math –a non-profit that provides PD and technical assistance to childcare providers on building play-based math for early learners and equipping families with tools to create math opportunities for children at home. Key lessons from both organizations include building relationships with DLL communities to understand and meet their needs, tailoring family outreach with a focus on educational equity, counting on parent leaders as effective advocates for DLL families, using asset-based, culturally relevant approaches, using play-based learning to support the development of math skills, and making materials accessible to DLL families (e.g., free, low-cost, in multiple languages). This review surfaces multiple gaps in research, practice, and policies for supporting early math learning for DLLs. Recommendations for policymakers and advocates include: (1) engaging and listening to DLL families to better understand their needs; (2) paying attention to specific needs of DLLs in COVID-19-related funding and policies; (3) including DLL-related reporting and capacity-building in early childcare quality ratings and improvement systems; (4) supporting grow-your-own bilingual educator certification pathways, and (5) supporting funding for educators to learn best practices while working with DLL students. Recommendations for practice include: (1) raising awareness about the importance of early math, (2) developing campaigns around early math targeted for DLL families, (3) developing and piloting training tools related to DLL instruction and family engagement, (4) investing in community organization efforts for families of DLLs around education equity, and (5) joining or supporting train the trainer professional development programs. Last, recommendations for research include: (1) conducting rapid-cycle research for effective distance learning for DLLs, (2) investigating the effectiveness of specific math instructional strategies for DLLs, and (3) refining training tools for DLL instruction based on research.

SOURCE: report *TYPE*: review/guidance
KEY WORDS: mathematics instruction, mathematics skills, young children, preschool education, early childhood education

Moschkovich, J. N. (2014). Bilingual/multilingual issues in learning mathematics. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 57–61). Springer.

This text provided an overview of some key issues, ideas, and findings surrounding bilingual and multilingual instruction and learning of mathematics. The author addressed a prevailing misconception that code-switching, a common practice among bilinguals where they switch languages during a sentence or conversation, is somehow a sign of deficiency. Researchers in linguistics agree that code-switching is not random or a reflection of language deficiency. Therefore, Moschkovich highlighted the criticality of not using someone's code-switching to reach superficial conclusions about their language proficiency, ability to recall a word, knowledge of a particular mathematics concept, or mathematical proficiency. Rather than viewing code-switching as a deficiency, the author asserted that instruction for bilingual mathematics learners should consider how this practice serves as a resource for communicating mathematically. Overall, Moschkovich demonstrated that there is strong evidence suggesting that bilingualism does not impact mathematical reasoning or problem solving.

SOURCE: book *TYPE*: review and guidance
KEY WORDS: mathematics, bilingual education, multilingualism, code-switching

Saxe, G. B., & Sussman, J. (2019). Mathematics learning in language inclusive classrooms: Supporting the achievement of English Learners and their English proficient peers. *Educational Researcher, 48*(7), 452–465. https://doi.org/10.3102/0013189X19869953

Inequities in mathematics learning opportunities between ELs and English proficient (EP) students have resulted in achievement gaps on national and state assessments. In this investigation, Saxe and Sussman reconsider a previous study to examine the effects of Learning Mathematics Through Representations (LMR), a data-driven, innovative curriculum intended to close the achievement gap between EL and EP students. The innovative LMR curriculum included 19-lessons on integers and fractions through use of a number line to explore mathematical ideas, construct arguments and elaborate explanations. Each lesson consisted of a five-phase structure that supported teachers to build student thinking and encourage math talk including coordinated use of visuals, manipulatives, encouraging explanation, and listening to fellow students' contributions. Saxe and Sussman conducted a multilevel analysis of California Standardized Test (CST) scores of students in LMR vs. comparison classrooms. To create the LMR group (n=11 classrooms) and a matched comparison group (n=10 classrooms), teachers were matched along three indicators: greatest degree, years of teaching experience, and previous professional development. The participants in this study included a total of 571 fourth and fifth grade students reflecting the ethnic/racial and socioeconomic diversity of three urban and suburban school districts in the San Francisco Bay Area. Findings revealed that EL students benefited more in LMR classrooms than in the comparison classrooms, that the growth in the achievement of EL students kept pace with the EP students in the same LMR classrooms, and that the EL-EP achievement gap was reduced when the achievement of EL students in LMR classrooms was compared to that of EP students in the comparison classrooms. The researchers attributed the positive outcomes for ELs on features of the LMR curriculum which engaged ELs in mathematics in ways that are not seen in published curriculum. Use of visual representations, verbal and written representations, sensorimotor, and five-phase lessons all supported teachers

to assess, elicit, and integrate student reasoning in classroom mathematical discussion and to adapt their instruction and meet students' needs. While this study did not examine which specific features of the LMR curriculum supported ELs the most, it did show that LMR as a whole, engaged ELs in ways that resulted in more equitable learning opportunities. This study is useful for researchers and for practitioners developing instructional approaches that engage all students with rich learning opportunities.

SOURCE: journal article *TYPE*: empirical
KEY WORDS: achievement gap, assessment, curriculum, diversity, elementary mathematics, learning environments

Song, K. H., & Coppersmith, S. A. (2020). Working toward linguistically and culturally responsive math teaching through a year-long urban teacher training program for English Learners. *Journal of Urban Mathematics Education, 1*3(2), 60–86. https://doi.org/10.21423/jume-v13i2a409

In this work, Song and Coopersmith sought to answer the following research question: How did participating urban in-service teachers apply linguistically and culturally responsive mathematics teaching competencies for ELs learned at a university EL teacher training program to their actual mathematics instruction in the classroom? The researchers employed a qualitative case study design and used data collected from observations and interviews with three teachers participating in a year-long in-service program for mathematics strategies for ELs. The three teachers were from two urban schools serving language diverse populations. Observational data showed that all three teachers in this study applied what they learned about linguistically and culturally responsive mathematics teaching in their classrooms. Observational data also showed that teachers did not specifically tailor their linguistically and culturally responsive teaching practices to each student, assuming instead that good strategies might work for all. Interviews with the teachers revealed that they felt constrained by personal, classroom, and school management structures (e.g., bell schedule). Despite the limitations of a small sample since, this study highlighted differences between what teachers learn in university-based professional development programs and what transfers into their actual practice. Song and Coopersmith suggest that in-service mathematics should prepare teachers more directly and explicitly for knowing when and how to develop and implement mathematical discourse.

SOURCE: journal article *TYPE*: empirical
KEY WORDS: activity theory, in-service teachers, teaching competencies, urban education

Zhao, M., & Lapuk, K. (2019). Supporting English learners in the math classroom: Five useful tools. *The Mathematics Teacher*, *112*(4), 288–293. https://doi.org/10.5951/mathteacher.112.4.0288

Zhao and Lapuk offer five proven strategies to support English Learners' math and language skills. The implementation of these tools is informed by the Sheltered Instruction Observation Protocol (SIOP). The first tool is building cultural background knowledge, i.e., getting to know the mathematical language difference and help English Learners (ELs) to connect our mathematical dialect with their own. Second, it is important to build vocabulary in mathematics because students benefit from attention to both language and content objectives, and because learning languages facilitates metacognitive skills development for information processing. Third, teachers should support reading with the use of engaging, relevant text in mathematics word problems (e.g., use culturally, linguistically, and developmentally appropriate situations). Fourth, teachers should support writing in mathematics via means such as a double-entry journal (i.e., students use their first language as the mediator between the math problem and their English explanation). Finally, teachers can support speaking by encouraging a free language environment for ELs (e.g., allowing them to take more time and choose the language they are comfortable with to solve math problems). These tools support bilingual learners' mathematics learning and teachers' creation of a multicultural and welcoming learning environment.

SOURCE: journal article *TYPE:* guidance
KEY WORDS: mathematics education, mathematics instruction, English language learners, mathematics teachers, secondary school teachers

Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). *Principles for the design of mathematics curricula: Promoting language and content development*. http://ell.stanford.edu/content/mathematics-resources-additional-resources

In order to provide support for linguistically and culturally diverse students who are learning English and mathematics simultaneously, Zwiers and colleagues offer mathematics teachers a framework for organizing strategies and considerations to facilitate students' progress in mathematics practices, content and language. The framework is based on the theory of action with four components, including the interdependency of language and disciplinary learning, the central role of student agency in linguistic and mathematic sense-making, the significance of scaffolding routines to foster students' autonomy, and the importance of instructional responsiveness to students' performance. Four principles guide the mathematics curriculum development and instruction for English language learners, including: (1) support students' sense-making with scaffolded tasks and amplified disciplinary language, (2) optimize output, i.e., increase the quality and quantity of opportunities for students to communicate their mathematical reasoning, (3) cultivate conversation, i.e., use lessons and activities to build a classroom culture that motivates and values communication, and (4) maximize meta-awareness (i.e., thinking about one's thought processes) by directing students' attention to what they need to do to improve mathematical reasoning and communication. The authors then present eight mathematical language routines that are most effective and practical for mathematical content and language learning. Mathematical language routines are structured but adaptive formats to amplify, develop. and assess students' language. The eight effective math language routines are: (1) stronger and clearer each time: to provide students with a purpose for communication and

strengthen output, (2) collect and display: to capture students' oral performance output into a stable reference, (3) critique, correct and clarify: to let students analyze, reflect on and improve on mathematical writing that is not their own, (4) information gap: to forge a need for communication, (5) co-craft questions and problems: to allow students to understand a context before producing answers and analyze how different situations are represented by different mathematical forms, (6) three reads: to ensure students' understanding of the texts/tasks and the presentations of the math questions and provide students with tools to negotiate meaning, (7) compare and connect: to promote students' meta-awareness, and (8) discussion supports: to promote enriching and inclusive discussions on math ideas and strategies. Zwiers and colleagues provide examples to accompany these mathematical language routines to illustrate how they can be enacted in practical classroom situations.

SOURCE: report *TYPE:* theoretical/guidance
KEY WORDS: linguistically and culturally diverse students, mathematical language development, curricular framework, design principles, mathematical language routines